1

Fundamentals of Pharmaceutical Calculations

Objectives

Upon successful completion of this chapter, the student will be able to:

- Convert common fractions, decimal fractions, and percentages to their corresponding equivalent expressions and apply each in calculations.
- Utilize exponential notations in calculations.
- Apply the method of ratio and proportion in problem-solving.
- Apply the method of dimensional analysis in problem-solving.
- Demonstrate an understanding of significant figures.

Pharmaceutical calculations is the area of study that applies the basic principles of mathematics to the preparation and safe and effective use of pharmaceuticals. Mathematically, pharmacy students beginning use of this textbook are well prepared. It is the *application* of the mathematics that requires study.

This initial chapter is intended to remind students of some previously learned mathematics. Most students likely will progress rapidly through this chapter. A student's performance on the practice problems following each section can serve as the guide to the degree of refreshment required.

Common and Decimal Fractions

Common fractions are portions of a whole, expressed at $\frac{1}{3}$, $\frac{7}{8}$, and so forth. They are used only rarely in pharmacy calculations nowadays. It is recalled, that when adding or subtracting fractions, the use of a *common denominator* is required. The process of multiplying and dividing with fractions is recalled by the following examples.

Examples:

If the adult dose of a medication is 2 teaspoonsful (tsp.), calculate the dose for a child if it is $\frac{1}{4}$ of the adult dose.

$$\frac{1}{4} \times \frac{2\text{ tsp.}}{1} = \frac{2}{4} = \frac{1}{2}\text{ tsp., } \textit{answer}$$

If a child's dose of a cough syrup is $\frac{3}{4}$ teaspoonful and represents $\frac{1}{4}$ of the adult dose, calculate the corresponding adult dose.

$$\frac{3}{4}\text{ tsp.} \div \frac{1}{4} = \frac{3}{4}\text{ tsp.} \times \frac{4}{1} = \frac{3 \times 4}{4 \times 1}\text{ tsp.} = \frac{12}{4}\text{ tsp.} = 3\text{ tsp., } \textit{answer}$$

NOTE: When common fractions appear in a calculations problem, it is often best to convert them to decimal fractions before solving.

A ***decimal fraction*** is a fraction with a denominator of 10 or any power of 10 and is expressed decimally rather than as a common fraction. Thus, $\frac{1}{10}$ is expressed as 0.10 and $\frac{45}{100}$ as 0.45. It is important to include the zero before the decimal point. This draws attention to the decimal point and helps eliminate potential errors. Decimal fractions often are used in pharmaceutical calculations.

To convert a common fraction to a decimal fraction, divide the denominator into the numerator. Thus, $\frac{1}{8} = 1 \div 8 = 0.125$.

To convert a decimal fraction to a common fraction, express the decimal fraction as a ratio and reduce.

Thus, $0.25 = \frac{25}{100} = \frac{1}{4}$ or ¼.

Arithmetic Symbols

Table 1.1 presents common arithmetic symbols used in pharmaceutical calculations.

Percent

The term ***percent*** and its corresponding sign, %, mean "in a hundred." So, *50 percent (50%)* means 50 parts in each one hundred of the same item.

TABLE 1.1 SOME ARITHMETIC SYMBOLS USED IN PHARMACY[a]

SYMBOL	MEANING
%	percent; parts per hundred
‰	per mil; parts per thousand
+	plus; add; or positive
−	minus; subtract; or negative
±	add or subtract; plus or minus; positive or negative; expression of range, error, or tolerance
÷	divided by
/	divided by
×	times; multiply by
<	value on left is less than value on right (e.g., 5<6)
=	is equal to; equals
>	value on left is greater than value on right (e.g., 6>5)
≠	is not equal to; does not equal
≈	is approximately equal to
≡	is equivalent to
≤	value on left is less than or equal to value on right
≥	value on left is greater than or equal to value on right
.	decimal point
,	decimal marker (comma)
:	ratio symbol (e.g., *a*:*b*)
::	proportion symbol (e.g., *a*:*b*::*c*:*d*)
∝	varies as; is proportional to

[a] Table adapted from *Barron's Mathematics Study Dictionary* (Barron's Educational Series, Inc. Hauppauge, NY: 1998.) by Frank Tapson with the permission of the author. Many other symbols (either letters or signs) are used in pharmacy, as in the metric and apothecaries' systems of weights and measures, in statistics, in pharmacokinetics, in prescription writing, in physical pharmacy, and in other areas. Many of these symbols are included and defined elsewhere in this text.

TABLE 1.2 EQUIVALENCIES OF COMMON FRACTIONS, DECIMAL FRACTIONS, AND PERCENT

COMMON FRACTION	DECIMAL FRACTION	PERCENT (%)	COMMON FRACTION	DECIMAL FRACTION	PERCENT (%)
$1/1000$	0.001	0.1	$1/5$	0.2	20
$1/500$	0.002	0.2	$1/4$	0.25	25
$1/100$	0.01	1	$1/3$	0.333	33.3
$1/50$	0.02	2	$3/8$	0.375	37.5
$1/40$	0.025	2.5	$2/5$	0.4	40
$1/30$	0.033	3.3	$1/2$	0.5	50
$1/25$	0.04	4	$3/5$	0.6	60
$1/15$	0.067	6.7	$5/8$	0.625	62.5
$1/10$	0.1	10	$2/3$	0.667	66.7
$1/9$	0.111	11.1	$3/4$	0.75	75
$1/8$	0.125	12.5	$4/5$	0.8	80
$1/7$	0.143	14.3	$7/8$	0.875	87.5
$1/6$	0.167	16.7	$8/9$	0.889	88.9

Common fractions may be converted to percent by dividing the numerator by the denominator and multiplying by 100.

Example:

Convert $3/8$ to percent.

$$3/8 \times 100 = 37.5\%, \textit{ answer.}$$

Decimal fractions may be converted to percent by multiplying by 100.

Example:

Convert 0.125 to percent.

$$0.125 \times 100 = 12.5\%, \textit{ answer.}$$

Examples of equivalent expressions of common fractions, decimal fractions, and percent are shown in Table 1.2. A useful exercise is to add to the content of this table by working with common fractions that are not listed (e.g., $1/400$, $1/60$, etc.).

PRACTICE PROBLEMS

1. How many 0.000065-gram doses can be made from 0.130 gram of a drug?
2. Give the decimal fraction and percent equivalents for each of the following common fractions:
   (a) $1/35$
   (b) $3/7$
   (c) $1/250$
   (d) $1/400$
3. If a clinical study of a new drug demonstrated that the drug met the effectiveness criteria in 646 patients of the 942 patients enrolled in the study, express these results as a decimal fraction and as a percent.
4. A pharmacist had 3 ounces of hydromorphone hydrochloride. He used the following:
   $1/8$ ounce
   $1/4$ ounce
   $1\frac{1}{2}$ ounces

   How many ounces of hydromorphone hydrochloride were left?

5. A pharmacist had 5 grams of codeine sulfate. He used it in preparing the following:

   8 capsules each containing 0.0325 gram
   12 capsules each containing 0.015 gram
   18 capsules each containing 0.008 gram

   How many grams of codeine sulfate were left after he had prepared the capsules?

6. The literature for a pharmaceutical product states that 26 patients of the 2,103 enrolled in a clinical study reported headache after taking the product. Calculate (a) the decimal fraction and (b) the percentage of patients reporting this adverse response.

Exponential Notation

Many physical and chemical measurements deal with either very large or very small numbers. Because it often is difficult to handle numbers of such magnitude in performing even the simplest arithmetic operations, it is best to use exponential notation or *powers of 10* to express them. Thus, we may express *121* as 1.21×10^2, *1210* as 1.21×10^3, and *1,210,000* as 1.21×10^6. Likewise, we may express *0.0121* as 1.21×10^{-2}, 0.00121 as 1.21×10^{-3}, and *0.00000121* as 1.21×10^{-6}.

When numbers are written in this manner, the first part is called the ***coefficient***, customarily written with one figure to the left of the decimal point. The second part is the ***exponential factor*** or *power of 10*.

The exponent represents the number of places that the decimal point has been moved—positive to the left and negative to the right—to form the exponential. Thus, when we convert *19,000* to 1.9×10^4, we move the decimal point 4 places to the left; hence the exponent 4. And when we convert *0.0000019* to 1.9×10^{-6}, we move the decimal point 6 places to the right; hence the *negative* exponent $^{-6}$.

In the *multiplication* of exponentials, the exponents are *added*. For example, $10^2 \times 10^4 = 10^6$. In the multiplication of numbers that are expressed in exponential form, the *coefficients* are multiplied in the usual manner, and then this product is multiplied by the power of *10* found by algebraically *adding* the exponents.

Examples:

$$(2.5 \times 10^2) \times (2.5 \times 10^4) = 6.25 \times 10^6, \text{ or } 6.3 \times 10^6$$
$$(2.5 \times 10^2) \times (2.5 \times 10^{-4}) = 6.25 \times 10^{-2}, \text{ or } 6.3 \times 10^{-2}$$
$$(5.4 \times 10^2) \times (4.5 \times 10^3) = 24.3 \times 10^5 = 2.4 \times 10^6$$

In the *division* of exponentials, the exponents are *subtracted*. For example, $10^2 \div 10^5 = 10^{-3}$. In the division of numbers that are expressed in exponential form, the *coefficients* are divided in the usual way, and the result is multiplied by the power of *10* found by algebraically *subtracting* the exponents.

Examples:

$$(7.5 \times 10^5) \div (2.5 \times 10^3) = 3.0 \times 10^2$$
$$(7.5 \times 10^{-4}) \div (2.5 \times 10^6) = 3.0 \times 10^{-10}$$
$$(2.8 \times 10^{-2}) \div (8.0 \times 10^{-6}) = 0.35 \times 10^4 = 3.5 \times 10^3$$

Note that in each of these examples, the result is rounded off to the number of *significant figures* contained in the *least* accurate factor, and it is expressed with only one figure to the left of the decimal point.

In the *addition* and *subtraction* of exponentials, the expressions must be changed (by moving the decimal points) to forms having any common power of 10, and then the coefficients only are added or subtracted. The result should be rounded off to the number of *decimal places* contained in the *least* precise component, and it should be expressed with only one figure to the left of the decimal point.

Examples:

$$(1.4 \times 10^4) + (5.1 \times 10^3)$$

$$\begin{aligned} & 1.4 \times 10^4 \\ 5.1 \times 10^3 = \; & \underline{0.51} \times 10^4 \\ \text{Total: } & 1.91 \times 10^4\text{, or } 1.9 \times 10^4\text{, } \textit{answer.} \end{aligned}$$

$$(1.4 \times 10^4) - (5.1 \times 10^3)$$

$$\begin{aligned} 1.4 \times 10^4 = \; & 14.0 \times 10^3 \\ & \underline{-5.1} \times 10^3 \\ \text{Difference: } & 8.9 \times 10^3\text{, } \textit{answer.} \end{aligned}$$

$$(9.83 \times 10^3) + (4.1 \times 10^1) + (2.6 \times 10^3)$$

$$\begin{aligned} & 9.83 \times 10^3 \\ 4.1 \times 10^1 = \; & 0.041 \times 10^3 \\ & \underline{2.6} \times \underline{10^3} \\ \text{Total: } & 12.471 \times 10^3\text{, or} \\ & 12.5 \times 10^3 = 1.25 \times 10^4\text{, } \textit{answer.} \end{aligned}$$

PRACTICE PROBLEMS

1. Write each of the following in exponential form:
   (a) 12,650
   (b) 0.0000000055
   (c) 451
   (d) 0.065
   (e) 625,000,000

2. Write each of the following in the usual numeric form:
   (a) 4.1×10^6
   (b) 3.65×10^{-2}
   (c) 5.13×10^{-6}
   (d) 2.5×10^5
   (e) 8.6956×10^3

3. Find the product:
   (a) $(3.5 \times 10^3) \times (5.0 \times 10^4)$
   (b) $(8.2 \times 10^2) \times (2.0 \times 10^{-6})$
   (c) $(1.5 \times 10^{-6}) \times (4.0 \times 10^6)$
   (d) $(1.5 \times 10^3) \times (8.0 \times 10^4)$
   (e) $(7.2 \times 10^5) \times (5.0 \times 10^{-3})$

4. Find the quotient:
   (a) $(9.3 \times 10^5) \div (3.1 \times 10^2)$
   (b) $(3.6 \times 10^{-4}) \div (1.2 \times 10^6)$
   (c) $(3.3 \times 10^7) \div (1.1 \times 10^{-2})$

5. Find the sum:
   (a) $(9.2 \times 10^3) + (7.6 \times 10^4)$
   (b) $(1.8 \times 10^{-6}) + (3.4 \times 10^{-5})$
   (c) $(4.9 \times 10^2) + (2.5 \times 10^3)$

6. Find the difference:
   (a) $(6.5 \times 10^6) - (5.9 \times 10^4)$
   (b) $(8.2 \times 10^{-3}) - (1.6 \times 10^{-3})$
   (c) $(7.4 \times 10^3) - (4.6 \times 10^2)$

Ratio, Proportion, and Variation

Ratio

The relative magnitude of two quantities is called their ***ratio***. Since a ratio relates the relative value of two numbers, it resembles a common fraction except in the way in which it is presented. Whereas a fraction is presented as, for example, $\frac{1}{2}$, a ratio is presented as 1:2 and is not read as "one half," but rather as "one is to two."

All the rules governing common fractions equally apply to a ratio. Of particular importance is the principle that *if the two terms of a ratio are multiplied or are divided by the same number, the value is unchanged,* the *value* being the quotient of the first term divided by the second. For example, the ratio *20:4* or $\frac{20}{4}$ has a value of *5;* if both terms are divided by *2,* the ratio becomes *10:2* or $\frac{10}{2}$, again the value of *5.*

The terms of a ratio must be of the same kind, for the *value* of a ratio is an abstract number expressing how many *times* greater or smaller the first term (or numerator) is than the second term (or denominator).[a] The terms may themselves be abstract numbers, or they may be concrete numbers of the same denomination. Thus, we can have a ratio of *20* to *4* ($\frac{20}{4}$) or *20 grams* to *4 grams* (20 grams/4 grams).

When two ratios have the same value, they are *equivalent.* An interesting fact about equivalent ratios is that the *product of the numerator of the one and the denominator of the other always equals the product of the denominator of the one and the numerator of the other; that is, the cross products are equal:*

$$\text{Because } \frac{2}{4} = \frac{4}{8},$$
$$2 \times 8 \text{ (or 16)} = 4 \times 4 \text{ (or 16)}.$$

It is also true that *if two ratios are equal, their reciprocals are equal:*

$$\text{Because } \frac{2}{4} = \frac{4}{8}, \text{ then } \frac{4}{2} = \frac{8}{4}.$$

We discover further that the *numerator of the one fraction equals the product of its denominator and the other fraction:*

$$\text{If } \frac{6}{15} = \frac{2}{5},$$
$$\text{then } 6 = 15 \times \frac{2}{5} \left(\text{or } \frac{15 \times 2}{5}\right) = 6,$$
$$\text{and } 2 = 5 \times \frac{6}{15} \left(\text{or } \frac{5 \times 6}{15}\right) = 2.$$

And the denominator of the one equals the quotient of its numerator divided by the other fraction:

$$15 = 6 \div \frac{2}{5} \left(\text{or } 6 \times \frac{5}{2}\right) = 15,$$
$$\text{and } 5 = 2 \div \frac{6}{15} \left(\text{or } 2 \times \frac{15}{6}\right) = 5.$$

An extremely useful practical application of these facts is found in *proportion.*

Proportion

A ***proportion*** is the expression of the equality of two ratios. It may be written in any one of three standard forms:

$$(1)\ a:b = c:d$$
$$(2)\ a:b :: c:d$$
$$(3)\ \frac{a}{b} = \frac{c}{d}$$

[a] The ratio of *1* gallon to *3* pints is not *1:3,* for the gallon contains *8* pints, and the ratio therefore is *8:3.*

Each of these expressions is read: *a is to b as c is to d,* and *a* and *d* are called the *extremes* (meaning "outer members") and *b* and *c* the *means* ("middle members").

In any proportion, *the product of the extremes is equal to the product of the means.* This principle allows us to find the missing term of any proportion when the other three terms are known. If the missing term is a *mean,* it will be *the product of the extremes divided by the given mean,* and if it is an *extreme,* it will be *the product of the means divided by the given extreme.* Using this information, we may derive the following fractional equations:

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then}$$
$$a = \frac{bc}{d},\ b = \frac{ad}{c},\ c = \frac{ad}{b}, \text{ and } d = \frac{bc}{a}.$$

In a proportion that is properly set up, the position of the unknown term does not matter. However, some persons prefer to place the unknown term in the fourth position—that is, in the denominator of the second ratio. *It important to label the units in each position (e.g., mL, mg) to ensure the proper relationship between the ratios of a proportion.*

The application of ratio and proportion enables the solution to many of the pharmaceutical calculation problems in this text and in pharmacy practice.

Examples:

If 3 tablets contain 975 milligrams of aspirin, how many milligrams should be contained in 12 tablets?

$$\frac{3 \text{ (tablets)}}{12 \text{ (tablets)}} = \frac{975 \text{ (milligrams)}}{x \text{ (milligrams)}}$$
$$x = \frac{12 \times 975}{3} \text{ milligrams} = 3900 \text{ milligrams, } \textit{answer.}$$

If 3 tablets contain 975 milligrams of aspirin, how many tablets should contain 3900 milligrams?

$$\frac{3 \text{ (tablets)}}{x \text{ (tablets)}} = \frac{975 \text{ (milligrams)}}{3900 \text{ (milligrams)}}$$
$$x = 3 \times \frac{3900}{975} \text{ tablets} = 12 \text{ tablets, } \textit{answer.}$$

If 12 tablets contain 3900 milligrams of aspirin, how many milligrams should 3 tablets contain?

$$\frac{12 \text{ (tablets)}}{3 \text{ (tablets)}} = \frac{3900 \text{ (milligrams)}}{x \text{ (milligrams)}}$$
$$x = 3 \times \frac{3900}{12} \text{ milligrams} = 975 \text{ milligrams, } \textit{answer.}$$

If 12 tablets contain 3900 milligrams of aspirin, how many tablets should contain 975 milligrams?

$$\frac{12 \text{ (tablets)}}{x \text{ (tablets)}} = \frac{3900 \text{ (milligrams)}}{975 \text{ (milligrams)}}$$
$$x = \frac{12 \times 975}{3900} \text{ tablets} = 3 \text{ tablets, } \textit{answer.}$$

Proportions need not contain whole numbers. If common or decimal fractions are supplied in the data, they may be included in the proportion without changing the method. For ease of calculation, it is recommended that common fractions be converted to decimal fractions prior to setting up the proportion.

Example:

If 30 milliliters (mL) represent $\frac{1}{6}$ of the volume of a prescription, how many milliliters will represent $\frac{1}{4}$ of the volume?

$$\tfrac{1}{6} = 0.167 \text{ and } \tfrac{1}{4} = 0.25$$

$$\frac{0.167 \text{ (volume)}}{0.25 \text{ (volume)}} = \frac{30 \text{ (mL)}}{x \text{ (mL)}}$$

$$x = 44.91 \text{ or} \approx 45 \text{ mL, } \textit{answer.}$$

Variation

In the preceding examples, the relationships were clearly *proportional.* Most pharmaceutical calculations deal with simple, *direct* relationships: twice the cause, double the effect, and so on. Occasionally, they deal with *inverse* relationships: twice the cause, half the effect, and so on, as when you *decrease* the strength of a solution by *increasing* the amount of diluent.[b]

Here is a typical problem involving inverse proportion:

If 10 pints of a 5% solution are diluted to 40 pints, what is the percentage strength of the dilution?

$$\frac{10 \text{ (pints)}}{40 \text{ (pints)}} = \frac{x\ (\%)}{5\ (\%)}$$

$$x = \frac{10 \times 5}{40}\% = 1.25\%, \textit{ answer.}$$

Dimensional Analysis

When performing pharmaceutical calculations, some students prefer to use a method termed ***dimensional analysis*** (also known as ***factor analysis***, ***factor-label method***, or ***unit-factor method***). This method involves the logical sequencing and placement of a series of ratios (termed ***factors***) into an equation. The ratios are prepared from the given data as well as from selected conversion factors and contain both arithmetic quantities and their units of measurement. Some terms are inverted (to their reciprocals) to permit the cancellation of like units in the numerator(s) and denominator(s) and leave only the desired terms of the answer. One advantage of using dimensional analysis is the consolidation of several arithmetic steps into a single equation.

In solving problems by dimensional analysis, the student unfamiliar with the process should consider the following steps:[1,2]

Step 1. Identify the given quantity and its unit of measurement.
Step 2. Identify the wanted unit of the answer.
Step 3. Establish the *unit path* (to go from the given quantity and unit to the arithmetic answer in the wanted unit), and identify the conversion factors needed. This might include:
(a) a conversion factor for the given quantity and unit, and/or
(b) a conversion factor to arrive at the wanted unit of the answer.
Step 4. Set up the ratios in the unit path such that cancellation of units of measurement in the numerators and denominators will retain only the desired unit of the answer.
Step 5. Perform the computation by multiplying the numerators, multiplying the denominators, and dividing the product of the numerators by the product of the denominators.

[b] In expressing an inverse proportion, we must not forget that *every* proportion asserts the equivalence of two ratios; therefore, the numerators must both be smaller or both larger than their respective denominators.

CALCULATIONS CAPSULE

Ratio and Proportion

- A *ratio* expresses the relative magnitude of two like quantities (e.g., 1:2, expressed as ''1 to 2.'')
- A *proportion* expresses the equality of two ratios (e.g., 1:2 = 2:4).
- The four terms of a proportion are stated as:

$$a{:}b = c{:}d, \text{ or, } a{:}b :: c{:}d, \text{ or } \frac{a}{b} = \frac{c}{d}$$

  and expressed as ''*a* is to *b* as *c* is to *d*.''
- Given three of the four terms of a proportion, the value of the fourth, or missing, term may be calculated.
- The ratio-and-proportion method is a useful tool in solving many pharmaceutical calculation problems.

The general scheme shown here and in the "Calculations Capsule: Dimensional Analysis" may be helpful in using the method.

Unit Path

Given Quantity	Conversion Factor for Given Quantity	Conversion Factor for Wanted Quantity	Conversion Computation	Wanted Quantity
				=

Example Calculations Using Dimensional Analysis

How many fluidounces (fl. oz.) are there in 2.5 liters (L)?

Step 1. The given quantity is *2.5 L.*
Step 2. The wanted unit for the answer is *fluidounces.*
Step 3. The conversion factors needed are those that will take us from liters to fluidounces. As the student will later learn, these conversion factors are:
1 liter = 1000 mL (to convert the given 2.5 L to milliliters), and
1 fluidounce = 29.57 mL (to convert milliliters to fluidounces)
Step 4. The unit path setup:

Unit Path

Given Quantity	Conversion Factor for Given Quantity	Conversion Factor for Wanted Quantity	Conversion Computation	Wanted Quantity
2.5 L	1000 mL	1 fl. oz.		=
	1 L	29.57 mL		

Note: The unit path is set up such that all units of measurement will cancel out except for the unit wanted in the answer, *fluidounces,* which is placed in the numerator.

Step 5. Perform the computation:

Unit Path

Given Quantity	Conversion Factor for Given Quantity	Conversion Factor for Wanted Quantity	Conversion Computation		Wanted Quantity
$2.5\,\cancel{L}$	$\frac{1000\,\cancel{mL}}{1\,\cancel{L}}$	$\frac{1\text{ fl. oz.}}{29.57\,\cancel{mL}}$	$\frac{2.5 \times 1000 \times 1}{1 \times 29.57} =$	$\frac{2500}{29.57}$	$= 84.55$ fl. oz.

or

$$2.5\,\cancel{L} \times \frac{1000\,\cancel{mL}}{1\text{ L}} \times \frac{1\text{ fl. oz.}}{29.57\,\cancel{mL}} = \frac{2.5 \times 1000 \times 1}{1 \times 29.57} = \frac{2500}{29.57} = 84.55\text{ fl. oz.}$$

Note: The student may wish to see the problem solved by ratio and proportion:

Step 1.

$$\frac{1\text{ (L)}}{2.5\text{ (L)}} = \frac{1000\text{ (mL)}}{x\text{ (mL)}};\ x = 2500\text{ mL}$$

Step 2.

$$\frac{29.57\text{ (mL)}}{2500\text{ (mL)}} = \frac{1\text{ (fl. oz.)}}{x\text{ (fl. oz.)}}$$

$x = 84.55$ fl. oz., *answer.*

A medication order calls for 1000 milliliters of a dextrose intravenous infusion to be administered over an 8-hour period. Using an intravenous administration set that delivers 10 drops/milliliter, how many drops per minute should be delivered to the patient?

Solving by dimensional analysis:

$$8\text{ hours} = 480\text{ minutes (min.)}$$

CALCULATIONS CAPSULE

Dimensional Analysis

- An alternative method to ratio and proportion in solving pharmaceutical calculation problems.
- The method involves the logical sequencing and placement of a series of ratios to consolidate multiple arithmetic steps into a single equation.
- By applying select conversion factors in the equation—some as reciprocals—unwanted units of measure cancel out, leaving the arithmetic result and desired unit.
- Dimensional analysis scheme:

Unit Path

Given Quantity	Conversion Factor for Given Quantity	Conversion Factor for Wanted Quantity	Conversion Computation	Wanted Quantity
				=

$$1000 \text{ mL} \times \frac{10 \text{ drops}}{1 \text{ mL}} \times \frac{1}{480 \text{ min.}} = 20.8 \text{ or } 21 \text{ drops per minute, } \textit{answer}.$$

Note: "Drops" was placed in the numerator and "minutes" in the denominator to arrive at the answer in the desired term, *drops per minute*.

The student may wish to see this problem solved by ratio and proportion:

Step 1.

$$\frac{480 \text{ (min.)}}{1 \text{ (min.)}} = \frac{1000 \text{ (mL)}}{x \text{ (mL)}}; \; x = 2.08 \text{ mL}$$

Step 2.

$$\frac{1 \text{ (mL)}}{2.08 \text{ (mL)}} = \frac{10 \text{ (drops)}}{x \text{ (drops)}}; \; x = 20.8 \text{ mL } \textit{or } 21 \text{ drops per minute, } \textit{answer}.$$

PRACTICE PROBLEMS

1. If an insulin injection contains 100 units of insulin in each milliliter, how many milliliters should be injected to receive 40 units of insulin?
2. Digoxin (LANOXIN) pediatric elixir contains 0.05 mg of digoxin in each milliliter of elixir. How many milligrams of digoxin would be administered with a dose of 0.6 mL?
3. In a clinical study, a drug produced drowsiness in 30 of the 1500 patients studied. How many patients of a certain pharmacy could expect similar effects, based on a patient count of 100?
4. A formula for 1250 tablets contains 6.25 grams (g) of diazepam. How many grams of diazepam should be used in preparing 350 tablets?
5. If 100 capsules contain 500 mg of an active ingredient, how many milligrams of the ingredient will 48 capsules contain?
6. Each tablet of TYLENOL WITH CODEINE contains 30 mg of codeine phosphate and 300 mg of acetaminophen. By taking two tablets daily for a week, how many milligrams of each drug would the patient take?
7. A cough syrup contains 10 mg of dextromethorphan hydrobromide per 5 mL. How many milligrams of the drug are contained in a 120-mL container of the syrup?
8. If an intravenous fluid is adjusted to deliver 15 mg of medication to a patient per hour, how many milligrams of medication are delivered per minute?
9. The biotechnology drug filgrastim (NEUPOGEN) is available in vials containing 480 micrograms (mcg) of filgrastim per 0.8 mL. How many micrograms of the drug would be administered by each 0.5 mL injection?
10. A prescription drug cost the pharmacist $42.00 for a bottle of 100 tablets. What would be the cost for 24 tablets?
11. How many 0.1-mg tablets will contain the same amount of drug as 50 tablets, each of which contains 0.025 mg of the identical drug?
12. Acyclovir (ZOVIRAX) suspension contains 200 mg of acyclovir in each 5 mL. How many milligrams of acyclovir are contained in a pint (473 mL) of suspension?
13. A metered dose inhaler contains 225 mg of metaproterenol sulfate, which is sufficient for 300 inhalations. How many milligrams of metaproterenol

sulfate would be administered in each inhalation?

14. A pediatric vitamin drug product contains the equivalent of 0.5 mg of fluoride ion in each milliliter. How many milligrams of fluoride ion would be provided by a dropper that delivers 0.6 mL?

15. If a pediatric vitamin contains 1500 units of vitamin A per milliliter of solution, how many units of vitamin A would be administered to a child given 2 drops of the solution from a dropper calibrated to deliver 20 drops per milliliter of solution?

16. An elixir contains 40 mg of drug in each 5 mL. How many milligrams of the drug would be used in preparing 4000 mL of the elixir?

17. An elixir of ferrous sulfate contains 220 mg of ferrous sulfate in each 5 mL. If each milligram of ferrous sulfate contains the equivalent of 0.2 mg of elemental iron, how many milligrams of elemental iron would be represented in each 5 mL of the elixir?

18. At a constant temperature, the volume of a gas varies inversely with the pressure. If a gas occupies a volume of 1000 mL at a pressure of 760 mm, what is its volume at a pressure of 570 mm?

19. If an ophthalmic solution contains 1 mg of dexamethasone phosphate in each milliliter of solution, how many milligrams of dexamethasone phosphate would be contained in 2 drops if the eyedropper used delivered 20 drops per milliliter?

20. A 15-mL package of nasal spray delivers 20 sprays per milliliter of solution, with each spray containing 1.5 mg of drug. *(a)* How many total sprays will the package deliver? *(b)* How many milligrams of drug are contained in the 15-mL package of the spray?

21. A penicillin V potassium preparation provides 400,000 units of activity in each 250-mg tablet. How many total units of activity would a patient receive from taking four tablets a day for 10 days?

22. If a 5-g packet of a potassium supplement provides 20 milliequivalents of potassium ion and 3.34 milliequivalents of chloride ion, (*a*) how many grams of the powder would provide 6 milliequivalents of potassium ion, and (*b*) how many milliequivalents of chloride ion would be provided by this amount of powder?

23. If a potassium chloride elixir contains 20 milliequivalents of potassium ion in each 15 mL of elixir, how many milliliters will provide 25 milliequivalents of potassium ion to the patient?

24. The blood serum concentration of the antibacterial drug ciprofloxacin increases proportionately with the dose of drug administered. If a 250-mg dose of the drug results in a serum concentration of 1.2 micrograms of drug per milliliter of serum, how many micrograms of drug would be expected per milliliter of serum following a dose of 500 mg of drug?

25. The dosage of the drug thiabendazole (MINTEZOL) is determined in direct proportion to a patient's weight. If the dose of the drug for a patient weighing 150 pounds is 1.5 grams, what would be the dose for a patient weighing 110 pounds?

26. If 0.5 mL of a mumps virus vaccine contains 5000 units of antigen, how many units would be present in each milliliter if the 0.5 mL of vaccine was diluted to 2 mL with water for injection?

27. A sample of Oriental ginseng contains 0.4 mg of active constituents in each 100 mg of powdered plant. How many milligrams of active constituents would be present in 15 mg of powdered plant?

Alligation

Alligation is an arithmetic method of solving problems relating mixtures of components of different strengths. There are two types of alligation: *alligation medial* and *alligation alternate*.

Alligation medial may be used to determine the strength of a common ingredient in a mixture of two or more preparations. For example, if a pharmacist mixed together known volumes of two or more solutions containing known amounts of a common ingredient, the strength of that ingredient in the resulting mixture can be determined by alligation medial.

Alligation alternate may be used to determine the proportion or quantities of two or more components to combine in order to prepare a mixture of a desired strength. For example, if a pharmacist wished to prepare a solution of a specified strength by combining two or more other solutions of differing concentrations of the same ingredient, the proportion or volumes of each solution to use may be determined by alligation alternate.

Alligation medial and alligation alternate may be used as options in solving a number of pharmaceutical calculations problems. The methods and problem examples are presented in Chapter 15.

Significant Figures

When we *count* objects accurately, *every* figure in the numeral expressing the total number of objects must be taken at its face value. Such figures may be said to be *absolute*. When we record a *measurement,* the last figure to the right must be taken to be an *approximation*, an admission that the limit of possible precision or of necessary accuracy has been reached and that any further figures to the right would not be significant—that is, either meaningless or, for a given purpose, needless.

A denominate number, like 325 *grams*, is interpreted as follows: The 3 means *300 grams,* neither more nor less, and the 2 means *exactly 20 grams more;* but the final 5 means *approximately 5 grams more*, i.e., *5 grams plus or minus some fraction of a gram*. Whether this fraction is, for a given purpose, negligible depends on how precisely the quantity was (or is to be) weighed.

Significant figures, then, are consecutive figures that express the value of a denominate number accurately enough for a given purpose. The accuracy varies with the number of significant figures, which are all absolute in value except the last, and this is properly called *uncertain.*

Any of the digits in a valid denominate number must be regarded as significant. Whether *zero* is significant, however, depends on its position or on known facts about a given number. The interpretation of *zero* may be summed up as follows:

CALCULATIONS CAPSULE

Significant Figures

- Digits other than zero are significant.
- A zero between digits is significant.
- Final zeros after a decimal point are significant.
- Zeros used only to show the location of the decimal point are not significant.

1. Any zero between digits is significant.
2. Initial zeros to the left of the first digit are never significant; they are included merely to show the location of the decimal point and thus give place value to the digits that follow.
3. One or more final zeros to the right of the decimal point may be taken to be significant.

Examples:

Assuming that the following numbers are all denominate:

1. In *12.5,* there are *three* significant figures; in *1.256, four* significant figures; and in *102.56, five* significant figures.
2. In *0.5,* there is *one* significant figure. The digit *5* tells us how many *tenths* we have. The nonsignificant *0* simply calls attention to the decimal point.
3. In *0.05,* there is still only *one* significant figure, as there is in *0.005.*
4. In *0.65,* there are *two* significant figures, and likewise *two* in *0.065* and *0.0065.*
5. In *0.0605,* there are *three* significant figures. The first *0* calls attention to the decimal point, the second *0* shows the number of places to the right of the decimal point occupied by the remaining figures, and the third *0* significantly contributes to the value of the number. In *0.06050,* there are *four* significant figures, because the final *0* also contributes to the value of the number.

One of the factors determining the degree of approximation to perfect measurement is the precision of the instrument used. It would be incorrect to claim that *7.76 milliliters* had been measured in a graduate calibrated in units of *1 milliliter,* or that *25.562 grams* had been weighed on a balance sensitive to 0.01 gram.

We must clearly distinguish *significant figures* from *decimal places.* When recording a measurement, the number of decimal places we include indicates *the degree of precision with which the measurement has been made,* whereas the number of significant figures retained indicates *the degree of accuracy* that is sufficient for a given purpose.

Sometimes we are asked to record a value "correct to (so many) decimal places." We should never confuse this familiar expression with the expression "correct to (so many) significant figures." For example, if the value *27.625918* is rounded to *five decimal places,* it is written *27.62592;* but when this value is rounded to *five significant figures,* it is written *27.626.*

Rules for Rounding

1. When rounding a measurement, retain as many figures as will give only one *uncertain* figure. For example, in using a ruler calibrated only in full centimeter units, it would be correct to record a measurement of 11.3 centimeters but not 11.32 centimeters, because the 3 (tenths) is uncertain and no figure should follow it.
2. When eliminating superfluous figures following a calculation, add 1 to the last figure retained in a calculation if it is 5 or more. For example, 2.43 may be rounded off to 2.4, but 2.46 should be rounded off to 2.5.
3. When adding or subtracting *approximate* numbers, include only as many decimal places as are in the number with the *fewest* decimal places. For example, when adding 162.4 grams + 0.489 grams + 0.1875 grams + 120.78 grams, the sum is 283.8565 grams, but the rounded sum is 283.9 grams. However, when an instrument has the capability to weigh precisely all the quantities in such a calculation, rounding may be deemed inappropriate.

   In this regard, *there is an assumption made in pharmaceutical calculations that all measurements in the filling of a prescription or in compounding a formula are performed with equal precision by the pharmacist.* Thus, for example, if the quantities 5.5 grams, 0.01 gram, and 0.005 gram are specified in a formula, they may be added as if they are precise weights, with a sum of 5.515 grams.

4. When multiplying or dividing two approximate numbers, retain no more significant figures than the number having the fewest significant figures. For example, if multiplying 1.6437 grams by 0.26, the answer may be rounded from the calculated 0.427362 grams to 0.43 grams.
5. When multiplying or dividing an approximate number by an absolute number, the result should be rounded to the same number of significant figures as in the approximate number. Thus, if 1.54 milligrams is multiplied by 96, the product, 243.84 milligrams, may be rounded to 244 milligrams, or to three significant figures.

PRACTICE PROBLEMS

1. State the number of significant figures in each of the *italicized* quantities:
   (a) One fluidounce equals *29.57* milliliters.
   (b) One liter equals *1000* milliliters.
   (c) One inch equals *2.54* centimeters.
   (d) The chemical costs *$1.05* per pound.
   (e) One gram equals *1,000,000* micrograms.
   (f) One microgram equals *0.001* milligram.
2. Round each of the following to three significant figures:
   (a) 32.75
   (b) 200.39
   (c) 0.03629
   (d) 21.635
   (e) 0.00944
3. Round each of the following to three decimal places:
   (a) 0.00083
   (b) 34.79502
   (c) 0.00494
   (d) 6.12963
4. If a mixture of seven ingredients contains the following approximate weights, what can you validly record as the approximate total combined weight of the ingredients?
   26.83 grams, 275.3 grams, 2.752 grams, 4.04 grams, 5.197 grams, 16.64 grams, and 0.085 gram.
5. Perform the following computations, and retain only significant figures in the results:
   (a) $6.39 - 0.008$
   (b) $7.01 - 6.0$
   (c) 5.0×48.3 grams
   (d) 24×0.25 gram
   (e) $56.824 \div 0.0905$
   (f) $250 \div 1.109$

Estimation

One of the best checks of the *reasonableness* of a numeric computation is an estimation of the answer. If we arrive at a wrong answer by using a wrong method, a mechanical repetition of the calculation may not reveal the error. But an absurd result, such as occurs when the decimal point is put in the wrong place, will not likely slip past if we check it against a preliminary estimation of what the result should be.

Because it is imperative that pharmacists ensure the accuracy of their calculations by every possible means, pharmacy students are urged to adopt *estimation* as one of those means. Proficiency in estimating comes only from constant practice. Therefore, pharmacy students are urged to acquire the habit of estimating the answer to every problem encountered before attempting to solve it. Estimation serves as a means for judging the reasonableness of the final result.

The estimating process is basically simple. First, the numbers given in a problem are *mentally* rounded off to slightly larger or smaller numbers containing fewer significant figures; for example, *59* would be rounded off to *60*, and *732* to *700*. Then, the required computations are performed, as far as possible *mentally,* and the result, although known to be somewhat greater or smaller than the exact answer, is close enough to serve as an estimate.

In *addition,* we can obtain a reasonable estimate of the total by first adding the figures in the leftmost column. The neglected remaining figures of each number are equally likely to express more or less than one-half the value of a unit of the order we have just added, and hence to the sum of the leftmost column we add half for every number in the column.

Example:

Add the following numbers: 7428, 3652, 1327, 4605, 2791, and 4490.
Estimation:
The figures in the thousands column add up to 21,000, and with each number on the average contributing 500 more, or every pair 1000 more, we get 21,000 + 3000 = 24,000, *estimated answer* (actual answer, 24,293).

In *multiplication,* the product of the two leftmost digits plus a sufficient number of *zeros* to give the right place value serves as a fair estimate. The number of *zeros* supplied must equal the total number of all discarded figures to the left of the decimal point. Approximation to the correct answer is closer if the discarded figures are used to round the value of those retained.

Example:

Multiply 612 by 413.
Estimation:
4 × 6 = 24, and because we discarded four figures, we must supply four zeros, giving 240,000, *estimated answer* (actual answer, 252,756).

In division, the given numbers may be rounded off to convenient approximations, but again, care is needed to preserve the correct place values.

Example:

Divide 2456 by 5.91.
Estimation:
The numbers may be rounded off to 2400 and 6. We may divide 24 by 6 mentally, but we must remember the two zeros substituted for the given 56 in 2456. The estimated answer is 400 (actual answer, 416).

PRACTICE PROBLEMS

1. Estimate the sums:

(a)	(b)	(c)
5641	3298	$75.82
2177	368	37.92
294	5192	14.69
8266	627	45.98
3503	4835	28.91
		49.87

2. Estimate the products:
   (a) 42 × 39 =
   (b) 365 × 98 =
   (c) 596 × 204 =
   (d) 6549 × 830 =
   (e) 8431 × 9760 =
   (f) 2.04 × 705.3 =
   (g) 0.0726 × 6951 =
   (h) 6.1 × 67.39 =

3. Estimate the quotients:
   (a) 171 ÷ 19 =
   (b) 184 ÷ 2300 =
   (c) 160 ÷ 3200 =
   (d) 86,450 ÷ 72 =
   (e) 98,000 ÷ 49 =
   (f) 1.0745 ÷ 500 =
   (g) 1.9214 ÷ 0.026 =
   (h) 458.4 ÷ 8 =

ANSWERS TO PRACTICE PROBLEMS

Common Fractions, Decimal Fractions, and Percent

1. 2000 doses
2. (a) 0.029 or 2.9%
   (b) 0.43 or 43%
   (c) 0.004 or 0.4%
   (d) 0.0025 or 0.25%
3. 0.69 or 69%
4. 1⅛ or 1.25 ounces hydromorphone hydrochloride
5. 4.416 grams codeine sulfate
6. 0.012 or 1.2%

Exponential Notations

1. (a) 1.265×10^4
   (b) 5.5×10^{-9}
   (c) 4.51×10^2
   (d) 6.5×10^{-2}
   (e) 6.25×10^8
2. (a) 4,100,000
   (b) 0.0365
   (c) 0.00000513
   (d) 250,000
   (e) 8,695.6
3. (a) $17.5 \times 10^7 = 1.75 \times 10^8$
   (b) $16.4 \times 10^{-4} = 1.64 \times 10^{-3}$
   (c) $6.0 \times 10^0 = 6.0$
   (d) $12 \times 10^7 = 1.2 \times 10^8$
   (e) $36 \times 10^2 = 3.6 \times 10^3$
4. (a) 3.0×10^3
   (b) 3.0×10^{-10}
   (c) 3.0×10^9
5. (a) 8.52×10^4, or 8.5×10^4
   (b) 3.58×10^{-5}, or 3.6×10^{-5}
   (c) 2.99×10^3, or 3.0×10^3
6. (a) 6.441×10^6, or 6.4×10^6
   (b) 6.6×10^{-3}
   (c) 6.94×10^3, or 6.9×10^3

Ratio, Proportion, Variation, and Dimensional Analysis

1. 0.4 mL insulin injection
2. 0.03 mg digoxin
3. 2 patients
4. 1.75 g diazepam
5. 240 mg
6. 420 mg codeine phosphate
   4200 mg acetaminophen
7. 240 mg dextromethorphan hydrobromide
8. 0.25 mg
9. 300 mcg filgrastim
10. $10.08
11. 12½ tablets
12. 18,920 mg acyclovir
13. 0.75 mg metaproterenol sulfate
14. 0.3 mg fluoride ion
15. 150 units vitamin A
16. 32,000 mg
17. 44 mg elemental iron
18. 1333 mL
19. 0.1 mg dexamethasone phosphate
20. (a) 300 sprays
    (b) 450 mg
21. 16,000,000 units
22. (a) 1.5 g
    (b) 1 milliequivalent chloride ion
23. 18.75 mL
24. 2.4 mc grams ciprofloxacin
25. 1.1 g thiabendazole
26. 2500 units antigen
27. 0.06 mg

Significant Figures

1. (a) four
   (b) four
   (c) three
   (d) three
   (e) seven
   (f) one
2. (a) 32.8
   (b) 200
   (c) 0.0363
   (d) 21.6
   (e) 0.00944
3. (a) 0.001
   (b) 34.795
   (c) 0.005
   (d) 6.130
4. 330.8 g
5. (a) 6.38
   (b) 1.0
   (c) 240 g
   (d) 6.0 g
   (e) 628
   (f) 225

Estimation

1. (a) 20,500 *(19,881)*
   (b) 14,500 *(14,320)*
   (c) $240.00 *($253.19)*
2. (a) 40 × 40 = 1600 *(1638)*
   (b) 360 × 100 = 36,000 *(35,700)*
   (c) 600 × 200 = 120,000 *(121,584)*
   (d) 7000 × 800 = 5,600,000 *(5,435,670)*
   (e) 8000 × 10,000 = 80,000,000 *(82,286,560)*
   (f) 2 × 700 = 1400 *(1438.812)*
   (g) (7 × 70) = 490 *(504.6426)*
   (h) 6 × 70 = 420 *(411.079)*
3. (a) 170 ÷ 20 = 8.5 *(9.0)*
   (b) 180 ÷ 2000 = 0.09 *(0.08)*
   (c) 16 ÷ 320 = 1/20 or 0.05 *(0.05)*
   (d) 8400 ÷ 7 = 1200 *(1200.7)*
   (e) 9800 ÷ 5 = 1960 *(2000)*
   (f) 0.01 ÷ 5 = 0.002 *(0.002149)*
   (g) 19 ÷ 0.25 = 19 × 4 = 76 *(73.9)*
   (h) 460 ÷ 8 = 57.5 *(57.3)*

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